Module/course code: module 7a/201400430
Date: April 17th 2015
Time: 8:45 – 10:15 (+25% for students who may use extra time)
Module-coördinator: Dr.ir. Ray Hueting
Instructor: Dr.ir. Ray Hueting, and Prof.dr. Jurriaan Schmitz

Type of test:
- Open questions

Allowed aids during the test:
- (Scientific) calculator

Attachments:
- Constants and formula sheet

Additional remarks:
- 2 Problems (PN junction, MOSFET) including scoring

- Important note: For speeding up the correction process please deliver your results of each problem on a separate set of papers!!!
**Name:** ______________________________
**St. nr:** ______________________________

**UNIVERSITY OF TWENTE.**

**Constants and equations sheet for Semiconductor Device Physics**

Elementary charge:
Thermal voltage equivalent (@ room temperature):
Dielectric constant (permittivity) Silicon:
Dielectric constant (permittivity) Silicon dioxide:
Intrinsic carrier concentration (if not given):
Electron diffusion constant (if not given):
Hole diffusion constant (if not given):
Electron mobility (if not given):
Hole mobility (if not given):

\[ q = 1.6 \times 10^{-19} \text{ C} \]
\[ u = kT/q = 0.025 \text{ V} \]
\[ \varepsilon_s = 10^{-12} \text{ F/cm} \]
\[ \varepsilon_{os} = 3.5 \times 10^{12} \text{ F/cm} \]
\[ n_i = \sqrt{2 \cdot 10^{10} \text{ cm}^3} \]
\[ D_n = 30 \text{ cm}^2/\text{s} \]
\[ D_p = 10 \text{ cm}^2/\text{s} \]
\[ \mu_n = 1200 \text{ cm}^2/\text{Vs} \]
\[ \mu_p = 350 \text{ cm}^2/\text{Vs} \]

1. **Semiconductor Physics** (spatially in one dimension)

**Fermi-Dirac distribution**

\[ f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \]

**Density of states (if not given)**

\[ g(E) \sim 10^{47} \sqrt{E} \]

**Carrier concentrations**

\[ n = N_c \exp\left(\frac{E_F - E_F}{kT}\right) = n_i \exp\left(\frac{E_F - E_F}{kT}\right) \]
\[ p = N_p \exp\left(\frac{E_F - E_F}{kT}\right) = n_i \exp\left(\frac{E_F - E_F}{kT}\right) \]

**Electrostatic potential**

\[ \psi = \frac{E_F}{q} \]

**Fermi potential**

\[ \varphi_F = \frac{E_F}{q} \]

**General formalism**

\[ n = n_i \exp\left(\frac{\psi - \varphi_F}{u_T}\right) \]
\[ p = n_i \exp\left(\frac{\varphi_F - \psi}{u_T}\right) \]

**Current equations**

\[ j_n = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx} = n\mu_n \frac{dE_{ FN}}{dx} \]
\[ j_p = qp\mu_p \mathcal{E} - qD_p \frac{dp}{dx} = p\mu_p \frac{dE_{ FP}}{dx} \]

**Einstein relation**

\[ D = u_T \cdot \mu = \frac{kT \cdot \mu}{q} \]

**Excess recombination rate (electrons)**

\[ R = \tilde{n}N_i c_n = \tilde{n}N_i v_n \sigma_n = \frac{\tilde{n}}{\tau_n} \]

**Continuity equation (electrons)**

\[ \frac{d\tilde{n}}{dt} = \frac{1}{q} \frac{dj_n}{dx} - (R - G) = D_n \frac{d^2\tilde{n}}{dx^2} - \frac{\tilde{n}}{\tau_n} \]
Excess carrier diffusion (electrons)

\[ \tilde{n}(x) = \tilde{n}_0 \exp\left( -\frac{x}{L_n} \right) \]
\[ L_n = \sqrt{D_n \tau_n} \]

Poisson's equation

\[ -\frac{d^2 \psi(x)}{dx^2} = \frac{d \mathcal{E}(x)}{dx} = \frac{\rho(x)}{\varepsilon_s} \]

2. pn junction

Built-in potential

\[ \phi_{bi} = u_r \ln \left( \frac{N_D N_A}{n_i^2} \right) \]

Depletion layer width

\[ W = \sqrt{\frac{2 \varepsilon_s (N_A + N_D)}{q N_A N_D}} (\phi_{bi} - V_A) \]

Junction current (Shockley eq.)

\[ I = A(j_n + j_p) = -Aq n_i^2 \left( \frac{D_n}{N_A I_n} + \frac{D_p}{N_D I_p} \right) \exp \left( \frac{V_A}{u_r} \right) - 1 \]

3. MOS transistor

Charge storage

\[ Q_n = -C_{ox}(V_{GB} - V_T) \]

Threshold voltage NMOS

\[ V_T = V_{FB} + (2\phi_B + V_{SB}) + \sqrt{\frac{2q \varepsilon_s N_A (2\phi_B + V_{SB})}{C_{ox}}} \]

Drain current NMOS (strong inversion)

\[ I_D = \frac{\mu_n C_{ox} W}{L} \left[ \left( V_{GS} - 2\phi_B - \frac{V_{DS}}{2} \right) V_{DS} - \frac{2}{3} \left( V_{SB} + 2\phi_B + V_{DS} \right)^{1.5} - \left( V_{SB} + 2\phi_B \right)^{1.5} \right] \]

Drain current NMOS (weak inversion)

\[ I_D = \frac{\mu_n W}{L} \int_0^L \frac{dQ_n}{dx} dx = \frac{\mu_n W}{L} Q_n = -\frac{\mu_n W u_f^2}{L} C_{dep} \exp \left( \frac{V_{GB} - \alpha \phi_B}{m \cdot u_f} \right) \]
PN-junction problems

The charge carrier distributions of a one-dimensional pn-junction are depicted in the figure below (linear scale). Both edges of the component have a metal contact (not shown) at a distance $L$ from the edges of the depletion layer.

For convenience sake we assume the following:
1. the junction operates at room temperature ($T=300$ K),
2. the doping concentration does not have any effect on the transport parameters such as the mobility and the bandgap,
3. the metal contacts of the component are “ohmic” (ideal),
4. the quasi-static approach is applicable for calculating the switching time,
5. and the depletion layers (the grey region in the figure) can be ignored in this question.

\[
\begin{align*}
\frac{p_n}{N_D} & = \frac{n_i^2}{e^{(V/\eta_T)}} \\
\frac{n_p}{N_A} & = \frac{n_i^2}{e^{(V/\eta_T)}} \\
\end{align*}
\]

\[
\begin{align*}
p_{n0} & = \frac{n_i^2}{N_D} \\
p_{p0} & = \frac{n_i^2}{N_A} \\
\end{align*}
\]

\[x \]

\[L \]

\[p,n \]

\[p,n \]

\[\text{a)} \quad \text{Is there recombination inside the (semiconductor) quasi-neutral regions? Explain.}
\bh) Do you think the device is forward biased? Explain.
\]

Let's assume that the applied bias $V$ is much higher than the thermal voltage ($V\gg \eta_T$) and we increase the doping concentration in the n-type region by a factor of five. The doping concentration in the p-type region is kept the same.

c) Sketch the charge carrier distributions again in linear scale for this situation.

d) Show that for the total current density now holds that $\approx \frac{J_n}{J} = -\frac{qD_n n_i^2}{N_A} \cdot \frac{V}{\eta_T}$. Do you think that $J$ has increased in comparison with the earlier situation (as shown in the figure above)? Explain.

For the transit (or delay) time we can write
\[
\tau = \frac{dQ}{dJ} = \frac{dQ}{dV} \cdot \frac{dV}{dJ} = C_{\text{diff}} \cdot \frac{dV}{dJ}
\]

where $Q$ is the total minority charge per unit area, and $C_{\text{diff}}$ is the diffusion capacitance per unit area.

e) Determine $Q$ in the device. Show that $C_{\text{diff}} = \frac{Q}{\eta_T}$.

Rating: a) 20 b) 10 c) 20 d) 25 e) 25 points
MOSFET problems

Question 1

The threshold voltage \( V_T \) of the MOS transistor is fine-tuned with an ion implantation of impurities (doping) in the channel. Explain how this influences both \( V_T \) and the ideality factor \( m \).

Question 2

Consider the graph below comprising 4 curves.

![Graph of Drain Current vs. Drain Voltage](image)

a) What is the qualitative difference between the curves? Explain.
b) Replicate the graph on your answer paper. Indicate the linear and saturation regimes in this figure.

c) How would you determine \( \beta (= \mu_n C_{ox} W / L) \) from a graph like this? Do you need additional information to quantify it?

Question 3

a) Make a cross section sketch of a MOS transistor in the linear region. Indicate the four terminals and other main features, and the charges that matter in the transistor action.
b) Derive the simple MOS drain current equation in the linear regime (as given below) from the fundamental physical action in the device.

\[
I_D = \beta \cdot \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS}
\]

c) Explain why the drain current of a MOS transistor does not go to 0 below the threshold voltage. Describe what happens instead.

Rating: Q1 25, Q2 a) 10 b) 10 c) 15, Q3 a) 10 b) 20 c) 10 points