Sample Exam
Quantitative Evaluation of Embedded Systems
(Winter 2020/2021)

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Student, Example

Name

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Student number Seat

Do not open this exam booklet before we ask you to. **Do** read this page carefully.

You can only write the exam at the seat allocated for you, and you must use the exam booklet that carries your name and student number.

This is a closed-book exam. The only material you may use is the hand-written A4 sheet that you submitted before the exam, and that you now find at your seat. Leave bags and jackets at a side of the room. Turn off any electronic devices and leave them with your bag. You may only take writing utensils, drinks, food, and your student or identity card to your seat. Please have your student or identity card clearly visible on your table.

Leaving the room without handing in your exam booklet is regarded as an attempt of deception. You may not leave the room during the first 30 minutes and the last 15 minutes of the exam. If you need to use the restrooms, please hand your exam booklet to the supervisor. Only one person at a time may leave for the restroom.

Before you start, please check that your exam booklet consists of **10 pages**, sequentially numbered, on 10 sheets of paper, and contains exercises 1 through 5 d). This is page 1.

Write your solutions on the (printed) right pages of the exam booklet, in the space provided below the respective questions. Solutions written in a different language than English, in red or similar colours, on the first or last page of the booklet, on the (blank) left pages, or on additional sheets not referenced from the booklet, **will not be graded**. Should you run out of space, ask the supervisor for an additional sheet of paper. You may use a pencil.

The duration of the exam is **90 minutes**. The total number of points of the exercises in this exam is 90. 45 points will be sufficient to pass the exam.

Have fun!

1	2	3	4	5	Sum
12	24	16	20	18	90

Question 1. True or False? (12 points)

For each of the following statements, decide whether it is true or false. If a statement is false, very briefly (i.e. in at most one sentence) state why or correct the statement.

a) In this exam, which is printed single-sided on A4 paper, only solutions written on the (blank) left pages will be graded.

b) A resit for the final exam of Quantitative Evaluation of Embedded Systems 2020/2021 will take place at the end of quarter 3.

c) The problem of determining whether a PCTL formula is satisfied for a (finite) LTS is decidable.

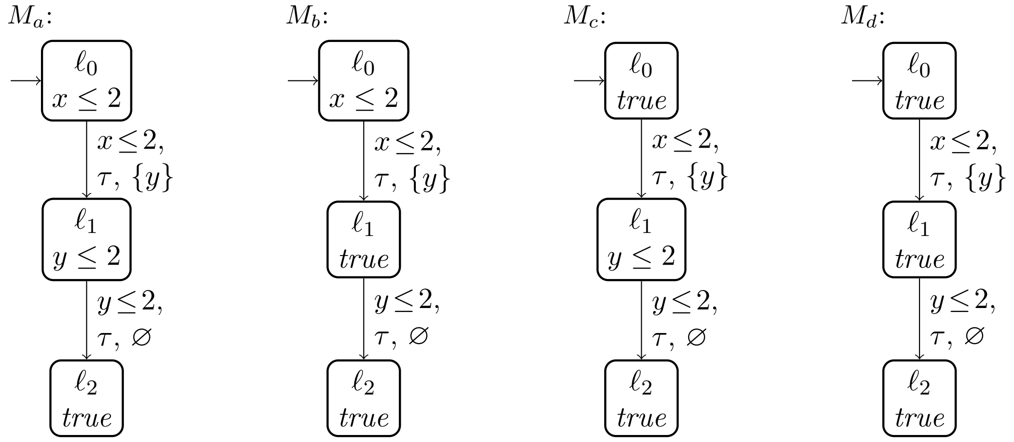
d) A discrete-time Markov chain is also a discrete-time stochastic process.

e) For a DTMC with n states, an implementation of value iteration needs to store $\mathcal{O}(\log n)$ values in memory at any time.

f) If we use the Okamoto bound for SMC and halve the error ϵ while keeping the confidence δ unchanged, then the necessary number of runs k at most doubles.

Question 2. Timed Automata (24 points)

Consider the following four TA M_a through M_d:



a) For each of these TA, give a TCTL formula that distinguishes it from the other three, i.e. a formula that is satisfied by that TA but not by the others. *(8 points)*

For M_a:

For M_b:

For M_c:

For M_d:

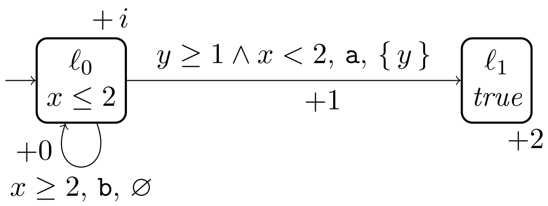
b) Let TA M_g be the same as M_a except that we replace "2" by "1" in all guards and invariants. Draw the digital clocks LTS of M_g. (It has 11 states.) *(8 points)*

Now consider the TA M_1 that has a single clock x, one location ℓ (which is thus also the initial location) with $inv(\ell) = x \leq 1$, and one edge $\ell \xrightarrow{true,\tau,\{x\}} \ell$.

c) Give a Zeno path with duration 1 in the semantics of M_1. *(3 points)*

d) Does this semantics contain a deadlock (i.e. a state s with $T(s) = \varnothing$)? *(1 point)*

Let us now work with the following TA M_2:



It is annotated with rewards: We have a rate reward of i in ℓ_0 and of 2 in ℓ_1, and an action reward of 1 on the edge labelled a out of ℓ_0. Let us refer to the corresponding reward structures as R_i, with $i \in \mathbb{N}$.

e) Give the value of $\mathrm{R}_{\min}(R_i, \{\ell_1\} \wedge x = 2)$, and a path that achieves this value, for the following values of i:

(i) $i = 1$: *(2 points)*

(ii) $i = 3$: *(2 points)*

Question 3. Hybrid Automata (16 points)

Consider a battery-powered system that can be connected to or disconnected from its charger, and that can be in use or idle. The connection and usage can change at any time, except that the system cannot be used when the battery is empty. The battery has a capacity of 10 000 J. The charger provides energy at a rate of 5 W. Using the system needs 4 W. (Recall that $1\,\mathrm{W} = 1\,\frac{\mathrm{J}}{\mathrm{s}}$.)

a) Draw a hybrid automaton modelling this system, using a linear battery model. Use continuous variable a to represent the battery's state of charge. Name locations according to the system configurations they represent, and use appropriate action labels to indicate the cause or result of each edge. *(8 points)*

b) What is the time unit of your model? *(1 point)*

c) What is the smallest subclass of HA that your model belongs to, of the subclasses you learned about in this course? Briefly justify your answer. *(3 points)*

d) Is your automaton initialised? *(1 point)*

e) In words or mathematical-style notation, give a (nontrivial) property about this system that can be model-checked using your automaton, i.e. that is decidable for the class of hybrid automata that your model belongs to. *(3 points)*

Question 4. Markov Chains and Decision Processes (20 points)

Consider a robot in a two-dimensional grid of 3×3 cells. Initially, the robot is in the corner cell $\langle 0, 0 \rangle$, and its goal is to move to the opposite corner cell $\langle 2, 2 \rangle$. In every cell, it can move either one cell forward in the x-direction (e.g. from cell $\langle 1, 1 \rangle$ to $\langle 2, 1 \rangle$) or one cell forward in the y-direction. It can never move backward (e.g. from cell $\langle 2, 1 \rangle$ to $\langle 2, 0 \rangle$). These are safe moves: they succeed with probability 1. Additionally, where this is possible without leaving the grid, the robot can also try to perform a move of two cells forward in x-direction and one cell forward in y-direction in one go. However, this is a risky move: it only succeeds with probability 0.1; with probability 0.9, it fails, and the robot does not move.

a) Draw the MDP representing the robot's world and its possible actions. Label your transitions with actions `x`, `y` and `d` for moves in x-, y- and diagonal direction, respectively. *(5 points)*

b) Use value iteration to compute $\mathrm{P}_{\max}(\Diamond^{\leq 3} \{ \langle 2, 2 \rangle \})$. *(5 points)*
Show your calculations in the usual tabular form.

c) Draw the DTMC induced by one scheduler of your choice that realises the maximum probability you computed in the previous question. Only include reachable states in your drawing. *(5 points)*

d) Decide whether the initial state of your robot world MDP satisfies the following PCTL formulas. Briefly explain your answers. *(5 points)*

(i) $\mathtt{P}_{>0.9}(\mathtt{F}\ \{\langle 2,2\rangle\})$:

(ii) $\mathtt{P}_{<1}(\neg\,\{\langle 0,1\rangle, \langle 2,1\rangle\}\ \mathtt{U}^{\leq 4}\ \{\langle 2,2\rangle\})$:

Question 5. Probabilistic Timed Automata (18 points)

Consider the following Modest PTA model:

```
clock x, y;
invariant(x <= 2) when(x >= 1) palt {
:1: {= y = 0 =}
:1: {= x = 0, y = 0 =}
};
invariant(y <= 5) stop
```

a) Draw the PTA represented by this model. *(4 points)*

b) Give the values for the following properties. For each value, briefly describe an optimal scheduler that achieves it. *(6 points)*

(i) $\mathrm{P}_{\mathrm{min}}(\diamond\, \mathsf{c}_2 \geq 4)$

(ii) $\mathrm{P}_{\mathrm{max}}(\diamond\, \mathsf{c}_2 \geq 4)$

(iii) $\mathrm{E}_{\mathrm{max}}(T, \mathsf{c}_1 \geq 4)$

c) Can we use the digital clocks technique to model-check probabilistic timed reachability properties on this model? Say why or why not. *(2 points)*

d) Draw the part of the region MDP of this PTA that includes all states where $y \leq 1$. Where a transition leaves this part, draw the transition (including its label), but write "..." for the successor state. You can write $fr(c)$ for the fractional part of the value of a clock c, i.e. $fr(c) = c - \lfloor c \rfloor$. *(6 points)*