Statistics & Probability (191506103)

Makeup Exam **(with 7 questions and 3 tables)**
Wednesday 23/01/19, 08:45 – 11:45

Full Marks : 35

1. Suppose U is a random variable having a Uniform distribution over the interval $(0, 1)$. Define a new r.v. $X := -\lambda^{-1} \ln(U)$, where $\lambda > 0$ is a constant. Determine the probability density function of X. Can you also recognize the distribution (by a descriptive name)? [4]

   [Hint: Recall that $\ln(x) < 0$, for $0 < x < 1$, and $\lim_{x \to 0} \ln(x) = -\infty$.]

2. Suppose that the random variables X and Y have the following joint probability density function:

$$f(x, y) = \frac{1}{2}\, xy, \qquad \text{for } 0 < x < y < 2 \qquad \text{and} \qquad = 0, \qquad \text{elsewhere.}$$

   (a.) Find the marginal pdf of Y. [2]

   (b.) Find the conditional pdf of X given $Y = y$ (where $0 < y < 2$).
   Calculate, also, $P(X > \frac{1}{4} \,|\, Y = \frac{1}{3})$. [2]

   (c.) Recall that $E(X|Y)$ is a random variable, which is a function of Y. Express the random variable in terms of Y. [2]

3. Based on previous experience, it is reasonable to assume that the chance of a driver getting a speeding ticket is 16%. A traffic police is checking the speeds of cars traveling on a road in a remote area. If during her stay at that place for a full hour, the officer sees 55 cars passing by, calculate the probability that the officer has to write more than 10 but less than 15 tickets in that hour. Answer as accurately as you can using the materials provided. [4]

   [Answers obtained directly from an advanced/graphical calculator would not be accepted.]

4. Recall that the moment generating function (mgf) of a $Poisson(\lambda)$ random variable is given by $\exp(-\lambda(1 - e^{-t}))$. Suppose X, Y and Z are three independent *Poisson* random variables with parameters λ, μ and ν, respectively. Identify the probability distribution of $U = X + Y + Z$ by first calculating the mgf of U. [Motivate your answer clearly.] [3]

5.(a.) Suppose that $\{X_1, X_2, \ldots, X_n\}$ is a random sample from a Bernoulli distribution with parameter θ $(0 < \theta < 1)$, i.e., X_i takes values 0 or 1 according to the probabilities

$$p(k; \theta) = P(X_i = k) = \theta^k (1 - \theta)^{(1-k)} \qquad k = 0, 1.$$

   Show that the maximum likelihood estimator of the parameter θ based on this sample is $\hat{\theta} = \bar{X} := \sum_{i=1}^{n} X_i / n$. [4]

   (b.) Show that the estimator $\hat{\theta}$ in part (a.) is unbiased for the parameter θ. [2]

6. For quality control purposes, twenty-five boxes of a certain brand of organic cornflakes are randomly selected and the sodium content (in miligram) in each box is measured. The sample average and standard deviation are found to be 129.89 and 1.08, respectively.

   (a.) Give an interval estimate for the expected sodium content of a randomly chosen box of organic cornflakes of the considered brand. Use a confidence level of 95%. Mention clearly any model assumption you make. [3]

   (b.) If you plan to obtain such interval estimates 5 more times, each time taking a new sample of 25 boxes, what is the probability that exactly 4 out of the 5 estimated intervals will contain the true (unknown) value of the expected sodium content? [2]

7. It is suspected that during promotion to the executive level in large corporations, the right-handed persons are preferred over the left-handed ones. From previous surveys it is known that about 85% of the general public is right-handed.

   (a.) A researcher has done a recent survey among 250 chief executive officers of large corporations and 224 of them are turned out to be right-handed. Does this provide enough evidence to conclude that the true percentage of right-handers among the executives in large corporations is higher than that among the general public? Use a significance level of $\alpha = 0.01$. [5]

   (b.) If the researcher included 25 executives instead of 250, would you be able to use the same procedure as used in part (a.)? Explain. [2]

$$\text{Final grade} = \left\{ \frac{\text{score on exam}}{35} \times 9 + 1 \right\} \text{ (rounded off to an integer)}$$