# Robust Control — EXAM

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| Course code: | 191560671 |
| Date: | 28-06-2016 |
| Time: | 08:45–11:45 (till 12:30 for students with special rights) |
| Course coordinator & instructor: | G. Meinsma |
| Type of test: | open book |
| Allowed aids during the test: | printed lecture notes, basic calculator |

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1. Determine $\|G_{y/u}\|_{\mathbb{H}_2}$ of the system $G_{y/u}$ described by

$$\dot{x}(t) = -x(t) + 2u(t)$$
$$y(t) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} x(t)$$

2. Consider

$$K_0(s) = \frac{B_0(s)}{A_0(s)} = 1, \qquad P(s) = \frac{s-1}{2-s}.$$

   (a) Does the controller $K_0$ make the closed loop asymptotically stable?

   (b) Is the closed loop with controller $K_0$ well posed?

   (c) With $K_0$ we can determine other controllers,

$$K = \frac{RD+Q}{-RN+Q}$$

   where $N(s) = s-1, D(s) = 2-s$ are the numerator and denominator of the plant. Under what conditions on the polynomials $R$ and $Q$ does this controller make the closed system asymptotically stable?

3. Suppose that the plant $P(s) = 1$ is controlled with a PI controller

$$K(s) = \frac{1}{2} + \frac{1}{s}.$$

   (a) Determine the phase margin

   (b) Determine the delay margin

4. Consider the standard unity feedback system with plant $P(s) = \frac{s-2}{(1-s)^2}$. Discuss the following problems:

   (a) For any stabilizing controller the step response of $H_{y/r}$ has overshoot?

   (b) For any stabilizing controller the step response of $H_{y/r}$ has undershoot?

   (c) What according to Freudenberg-Looze are "acceptable" closed loop bandwidths?

5. *Chapter 6:* Consider the LQ problem with

$$\begin{bmatrix} \dot{x} \\ z \end{bmatrix} = \left(\begin{array}{c|c} 1 & 1 \\ \hline 1 & 0 \\ 0 & \frac{1}{\sqrt{15}} \end{array}\right) \begin{bmatrix} x \\ u \end{bmatrix}$$

   Determine the solution $P$ of the corresponding LQ-Riccati equation and determine the LQ-optimal state feedback $u = -Fx$.

6. *Chapter 7:* Consider a family of first order polynomials

$$X(s) = as + b, \qquad a \in [\underline{a}, \overline{a}],\ b \in [\underline{b}, \overline{b}]$$

   with $\underline{a}, \overline{a}, \underline{b}, \overline{b}$ given numbers.

   (a) What does the Kharitonov test say for these first order polynomials?

   (b) Prove that the Kharitonov test is correct for these first order polynomials.

7. Consider the $\mathbb{H}_\infty$ filtering problem with

$$G_{m/w}(s) = \frac{-1}{s+2}, \qquad G_{y/w}(s) = \frac{s-3}{s+10}$$

   Solve the $\mathbb{H}_\infty$ filtering problem. Provide the optimal $K(s)$, the optimal $H_{z/w}(s)$ and optimal norm $\|H_{z/w}\|_{\mathbb{H}_\infty}$.

8. Exercise 9.2 of the lecture notes

| problem: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| points: | 4 | 2+2+2 | 2+3 | 2+2+2 | 4 | 2+2 | 4 | 1+2 |

Grade: $= 1 + 9\frac{p}{p_{\max}}$ (possibly with homework correction of $\leq 0.6$)