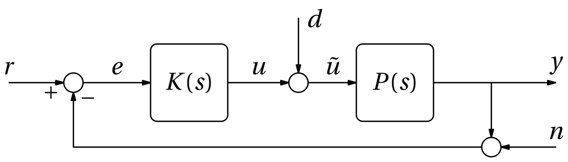
Robust Control — based on an 2014 exam

1. Consider the feedback scheme



and assume the closed loop is stable with both P and K proper rational transfer functions. Now instead of constant disturbances assume that the disturbance is sinusoidal,

$$d(t) = \cos(\omega_0 t), \qquad r(t) = 0, \quad n(t) = 0$$

for some fixed $\omega_0 \geq 0$. (This is, for instance, common in helicoptors due to rotating blades.) We say that the system *rejects* the disturbance if $y(t) = 0$ despite the presence of the disturbance.

   (a) Show that the system rejects the disturbance *for every stabilizing* $K(s)$ if and only if $P(\mathrm{i}\omega_0) = 0$

   (b) Suppose now $P(\mathrm{i}\omega_0) \neq 0$. Under which conditions on the controller $K(s)$ does the system reject the disturbance?

2. Suppose plant P and controller K are strictly proper and stable and that

$$\Re(P(\mathrm{i}\omega)) > 0 \quad \text{and} \quad \Re(K(\mathrm{i}\omega)) > 0$$

for all frequencies $\omega \in \mathbb{R}$. Show that the unity feedback system is stable.

3. (MATLAB EXERCISE REPLACED WITH): Determine the $\mathbb{H}_2$-norm of the system with transfer function $\frac{2}{s+2}$

4. (MATLAB EXERCISE REPLACED WITH): Exercise 6.6(b-d) for a given state representation of the system:

$$\dot{x} = -x + w, \quad z = x - u, \quad y = -2x + w$$

5. Consider the usual unity feedback scheme. Suppose that $P(s) = \dfrac{\rho + \Delta(s)}{s}$ with $\rho > 0$ constant, and that the constant controller $K(s) = 1$ is used.

   (a) Determine interconnection matrix $H(s)$ for this $\Delta(s)$.

   (b) For which constant ρ is the system stable for all stable Δ with $\|\Delta\|_\infty \leq 1$?

   (c) If ρ violates the above condition, does there then exist a *constant* $\Delta(s) = \delta$ with $\|\Delta\|_\infty < 1$ that destabilizes?

6. Problem 9.4(c)

problem:	1	2	3	4	5	6
points:	2+2	3	2	2+2+2	2+2+2	3

Grade: $= 1 + 9\frac{p}{p_{\max}}$