**UNIVERSITEIT TWENTE**
Faculteit Elektrotechniek, Wiskunde en Informatica

**Exam Random Signals and Filtering (201200135) on Tuesday April 14, 2015, 13.45 - 16.45 hours.**

The solutions of the exercises need to be clearly formulated and written in a well-structured manner. Moreover, you always need to present a derivation or arguments to support your answer.

You can use one single-sided A4 page of handwritten notes with your exam.

---

1. Consider $\Omega = [0, \infty)$ and let $\mathcal{P}$ be such that

$$\mathcal{P}([n, n+1)) = \frac{1}{2^{n+1}}, \quad \mathcal{P}([n, n+1]) = \frac{3}{2^{n+2}},$$

for $n = 0, 1, 2, \ldots$. Can $\mathcal{P}$ satisfy all axioms of a probability measure?

2. Consider the following nonlinear system:

$$\begin{aligned} X_{k+1} &= X_k^2 - W_k^2 \\ Y_k &= X_k V_k \end{aligned}$$

where $X_0$, $V_k$ and $W_k$ are mutually independent and all have a uniform distribution on the interval $[0, 1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine $E[X_0|Y_0]$

   b) Determine $E[X_1|Y_0]$.

3. Consider the following nonlinear system:

$$\begin{aligned} X_{k+1} &= X_k^2 + W_k \\ Y_k &= X_k^2 + V_k \end{aligned}$$

where $X_0$, $V_k$ and $W_k$ are mutually independent and all have a Gaussian distribution with mean 0 and variance 1. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine $E_{\text{lin}}[X_1|Y_0]$.

   b) Determine $E_{\text{lin}}[X_1|Y_0, Y_1]$.

see reverse side

Exam Random Signals and Filtering (201200135) on Tuesday April 14, 2015, 13.45 - 16.45 hours.

---

4. Consider the following nonlinear system:

$$X_{k+1} = (X_k - W_k)^2$$
$$Y_k = (X_k + V_k)^2$$

where $X_0$, $V_k$ and $W_k$ are mutually independent and all have a uniform distribution on $[-1, 1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

We are applying a particle filter without resampling where we recursively update our particles according to:

$$\pi(x_k \mid x_{k-1}^i, y_k) = p(x_k \mid x_{k-1}^i)$$

a) Clarify how to compute updated weights when a measurement arrives in a program such as Matlab (the algorithm not the precise Matlab code)

b) Clarify how to obtain updated particles $x_k^i$ given $x_{k-1}^i$ in a program such as Matlab (the algorithm not the precise Matlab code).

After step $k$ we have particles $x_k^i$ and associated weights $q_k^i$ to estimate $X_k$ based on measurements $y_0, \ldots, y_k$.

b) Indicate how to use the particles and weights to find an unbiased estimate for

$$E\left(X_k - E[X_k | y_0, \ldots, y_k]\right)^2$$

---

You can earn the following number of points for each exercise:

| | | | | | |
|---|---|---|---|---|---|
| Exercise | 1. | 3 points | Exercise | 2. | 5 points |
| Exercise | 4. | 5 points | Exercise | 5. | 5 points |

The grade is determined by adding two points to the total number of points and dividing by two.