UNIVERSITEIT TWENTE
Faculteit Elektrotechniek, Wiskunde en Informatica

Exam Random Signals and Filtering (201200135) on Monday July 1, 2013, 13.45 – 16.45 hours.

The solutions of the exercises need to be clearly formulated and written in a well-structured manner. Moreover, you always need to present a derivation or arguments to support your answer.

You can use one single-sided A4 page of handwritten notes with your exam.

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1. Consider $\Omega = \mathbb{R}$ and let $\mathcal{P}$ be such that

   $$\mathcal{P}((n, n+1)) = 0$$

   for all integers n. Show that

   $$\mathcal{P}([a, b]) = 0$$

   cannot be a continuous function for all $a, b \in \mathbb{R}$.

2. Consider the following nonlinear system:

   $$\begin{aligned} X_{k+1} &= W_k \cdot \sqrt{X_k} \\ Y_k &= X_k \cdot \sqrt{V_k} \end{aligned}$$

   where X_0, V_k and W_j are mutually independent for all k and j, and all have a uniform distribution on the interval $[0, 1]$. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine the density function associated to the stochastic variable X_1

   b) Determine $E[X_0 | Y_0]$

   c) Determine $E[X_2 | Y_1]$.

3. Consider the following nonlinear system:

   $$\begin{aligned} X_{k+1} &= X_k^2 + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

   where X_0, V_k and W_j are mutually independent for all k and j and all have a Gaussian distribution with mean 0 and variance 1. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

   a) Determine $E_{\text{lin}}[X_1 | Y_0]$

   b) Determine $E_{\text{lin}}[X_1 | Y_0, Y_1]$

z.o.z.

4. Consider the following linear system:

$$\begin{aligned} X_{k+1} &= X_k + W_k \\ Y_k &= X_k + V_k \end{aligned}$$

where X_0, V_k and W_k are mutually independent and all have a Gaussian distribution with mean zero and variance 1. Moreover, the noise sequences $\{W_k\}$ and $\{V_k\}$ are assumed to be white.

We are applying a particle filter without resampling where we recursively update our particles according to:

$$\pi(x_k \mid x_{k-1}^i, y_k) = p(x_k \mid y_k)$$

a) Clarify how you could implement the updating of particles in a program such as Matlab (the algorithm not the precise Matlab code)

b) Compute $\operatorname{var}(X_k^i - X_k)$.

c) Argue whether the performance of this particle filter would improve with resampling

You can earn the following number of points for each exercise:

Exercise	1.	2 points	Exercise	2.	5 points
Exercise	4.	5 points	Exercise	5.	6 points

The grade is determined by adding two points to the total number of points and dividing by two.