Part 1: Multiple Choice Questions

23 June 2020

Read carefully. There is no limit in the number of right or wrong answers.

Twist and Wrenches

1. Since $so(3)$ is a Lie algebra, it possesses an antisymmetric operator called the commutator given by $[\tilde{\omega}_1, \tilde{\omega}_2] := \tilde{\omega}_1\tilde{\omega}_2 - \tilde{\omega}_2\tilde{\omega}_1; \quad \tilde{\omega}_1, \tilde{\omega}_2 \in so(3)$. What is true about the commutator?
   - ◯ $[\tilde{\omega}_1, \tilde{\omega}_2] \in so(3)$
   - ◯ $[\tilde{\omega}_1, \tilde{\omega}_2] \in so^*(3)$
   - ◯ $[\tilde{\omega}_1, \tilde{\omega}_2] = 0$ implies that either $\omega_1 = 0$ or $\omega_2 = 0$
   - ◯ $[\tilde{\omega}, \tilde{\omega}] = 0$ if and only if $\tilde{\omega} = 0$
   - ◯ The commutator of its arguments in tilde form represents the scalar product of its arguments in vector form.
   - ◯ $[\tilde{\omega}_1, \tilde{\omega}_2] = 0$ implies that $\tilde{\omega}_1$ and $\tilde{\omega}_2$ commute.

2. Consider a rotating rigid body that is pinned from its center. Attached to the rotating body is the body-fixed frame Ψ_B. Now consider another fixed-intertial frame Ψ_I with the same origin as Ψ_B. The configuration of the body is described by R_B^I. Let p be a point attached to the rotating body. What is the velocity of p expressed in Ψ_I?
   - ◯ $p^i = \omega_I^{B,B} \wedge p^i$
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3. Consider the Lie group $SE(3)$. What is true about the space $T_H^*SE(3)$, that is the cotangent space on $SE(3)$ at the configuration $H \in SE(3)$?
   - ◯ There is a duality pairing between a twist $T \in se(3)$ and an element of $T_H^*SE(3)$, and the resulting scalar represents mechanical power along a rigid body motion.
   - ◯ It is a 6-dimensional vector space.
   - ◯ It has a natural inner product.
   - ◯ An element of $T_H^*SE(3)$ can be naturally paired with the vector $\dot{H}$ and the resulting scalar represents mechanical power along a rigid body motion.
   - ◯ It can be made into a Lie Group with a proper change of coordinates.
   - ◯ It is the dual space of the space $T_HSE(3)$.

Kinematics and Dynamics

1. The matrix exponential is a map from $so(3)$ to $SO(3)$. If we want to obtain $R_A^B(t) = e^{Xt}R_A^B(0)$ what should be in the place of X?
   - ◯ $\tilde{\omega}_A^{B,B}$
   - ◯ $T_A^{A,A}$
   - ◯ $\tilde{T}_A^{B,A}$
   - ◯ $\omega_B^{A,B}$
   - ◯ W^b
   - ◯ $\dot{H}_A^B$
2. The Euler-Lagrange equations in Robotics:
   - ◯ are an alternative way to derive the equations with respect to the least action principle.
   - ◯ are equations on the space of generalised forces on the configuration manifold T^*Q.
   - ◯ are equations of the space of generalised velocities on the configuration manifold TQ
   - ◯ Are n, with n the dimension of the configuration manifold Q, i.e. the DoFs of the Robot.
3. The Geometric Jacobian:
   - ◯ Does not depend on the configuration of the robot.
   - ◯ Contains the expression of the unit twists which characterise the kinematic pairs of the robot, expressed in the base frame Ψ_0.
   - ◯ has always a non null determinant if the robot is redundant.
   - ◯ May loose rank in some configurations.
   - ◯ Is a $(0-2)$ tensor.
   - ◯ Represents a linear transformation from TQ to $se(3)$
4. The inertia tensor $\mathcal{I}$ of a rigid body:
   - ◯ Is a linear transformation from $se(3)$ to $se(3)$
   - ◯ Is a $(0-2)$ tensor.
   - ◯ Is constant only in frames which are fixed with the rigid body.
   - ◯ Is diagonal in any body fixed frame.
   - ◯ Is constant only in the Principal Inertia Frame.
   - ◯ In case the rigid body reduces to a point it just represents the mass of the resulting point mass.

Control

1. Inverse dynamic control is a classical technique for position control based on feedback linearisation.
   - ◯ It is a control scheme that falls into the robust control techniques, i.e. it is robust with respect to small uncertainties of the parametric values of the system matrices M, C, G.
   - ◯ If applied in order to track a reference designed in the work-space, needs an off-line stage in which inverse dynamics is computed in order to find the corresponding trajectory in the joint-space.
   - ◯ Requires a perfect knowledge of the system matrices M and C, but not of G, since gravity compensation is taken care by the pole placement.
   - ◯ Is very efficient from a computational point of view, especially when interaction takes place.

- ◯ Presents an inner feedback linearisation loop, where the robot is dynamically transformed into a double integrator, and an external linear feedback loop which applies pole placement to stabilise a desired trajectory.
- ◯ Would not work if the inertia matrix were not positive definite for any configuration.

2. If the Geometric Jacobian of a robot arm is given as $J(q) : TQ \to se(3); \dot{q} \to T_n^{0,0}$ how can you calculate the torque or force that is felt at the joints, τ, due to an external wrench $W^{0,n}$ acting on the end-effector and expressed in the base frame Ψ_0? (*Keep in mind here τ and W are represented as row vectors.*)
   - ◯ $\tau^T = J(q)^T \mathrm{Ad}^T_{H_n^0} (W^{0,n})^T$
   - ◯ $\tau^T = J(q)^T (W^{0,n})^T$
   - ◯ $\tau^T = \mathrm{Ad}^T_{H_n^0} W^{0,n}$
   - ◯ $\tau^T = J(q)^T \mathrm{Ad}^T_{H_0^n} (W^{0,n})^T$

Part 3: Theory Lab

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Consider the topological space $(\mathbb{R}, \mathcal{O}_s)$ where $\mathcal{O}_s$ denotes the standard topology.

1. Show that $(\mathbb{R}, \mathcal{O}_s)$ is a one-dimensional topological manifold.
2. Provide a smooth atlas $\mathcal{A}$ for $(\mathbb{R}, \mathcal{O})$ that contains the smallest possible number of charts.
3. Check whether the set

$$\mathcal{A}' := \{((-\infty, 1), x), ((-1, \infty), y)\}$$

defines a smooth atlas for $(\mathbb{R}, \mathcal{O}_s)$, if the two chart maps are given by

$$x : (-\infty, 1) \to \mathbb{R}\,, \quad a \mapsto a \qquad \text{and} \qquad y : (-1, \infty) \to \mathbb{R}\,, \quad a \mapsto a^3\,.$$

4. Calculate the components of the tangent vector $X_{\gamma,7}$ to the curve

$$\gamma : \mathbb{R} \to \mathbb{R}\,, \qquad \lambda \mapsto 7e^{\lambda}$$

with respect to the chart $((6, 8), z)$, which shall be assumed to lie in some smooth atlas $\mathcal{A}''$ for $(\mathbb{R}, \mathcal{O}_s)$ and whose chart map is given by

$$z : (6, 8) \to \mathbb{R}\,, \qquad a \mapsto a^2\,.$$

5. Calculate $X_{\gamma,7}f$ for the function

$$f : \mathbb{R} \to \mathbb{R}\,, \qquad a \mapsto \sin(a)$$

on the smooth manifold $(\mathbb{R}, \mathcal{O}_s, \mathcal{A}'')$.

Part 3: Open Question

23 June 2020

Consider the dynamic equation (1) of a manipulator not touching the environment and ideally without friction, with mass matrix $M(q)$, $C(q,\dot{q})$ modelling the Coriolis and centrifugal forces, and $G(q)$ the gravitational forces due to the potential energy $V(q)$; together with a control law of the form (2).

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad , \quad G(q) := \frac{\partial V}{\partial q}(q) \tag{1}$$

$$\tau = -\frac{\partial V_c}{\partial q}(q) - B(q)\dot{q} \quad , \quad B(q) \geq 0 \tag{2}$$

A) Find the potential function $V_c(q)$ for which the manipulator behaves as if pulled towards q_d, some known reference trajectory in joint space, by a spring with positive definite stiffness matrix K—expressed in joint space, with an energy minimum at q_d.

B) Keep on considering no interaction with the environment. Discuss the main differences between the former *energy shaping* approach and and other position control technique in the joint-space: the *inverse dynamics control*.
What method is heavier from a computation point of view and why?

C) Discuss how to extend the energy shaping approach if the desired configuration is in the work-space instead of the joint-space. Discuss *if* and *why* you need to invert the geometric Jacobian in this case.

Part 4: Exercise

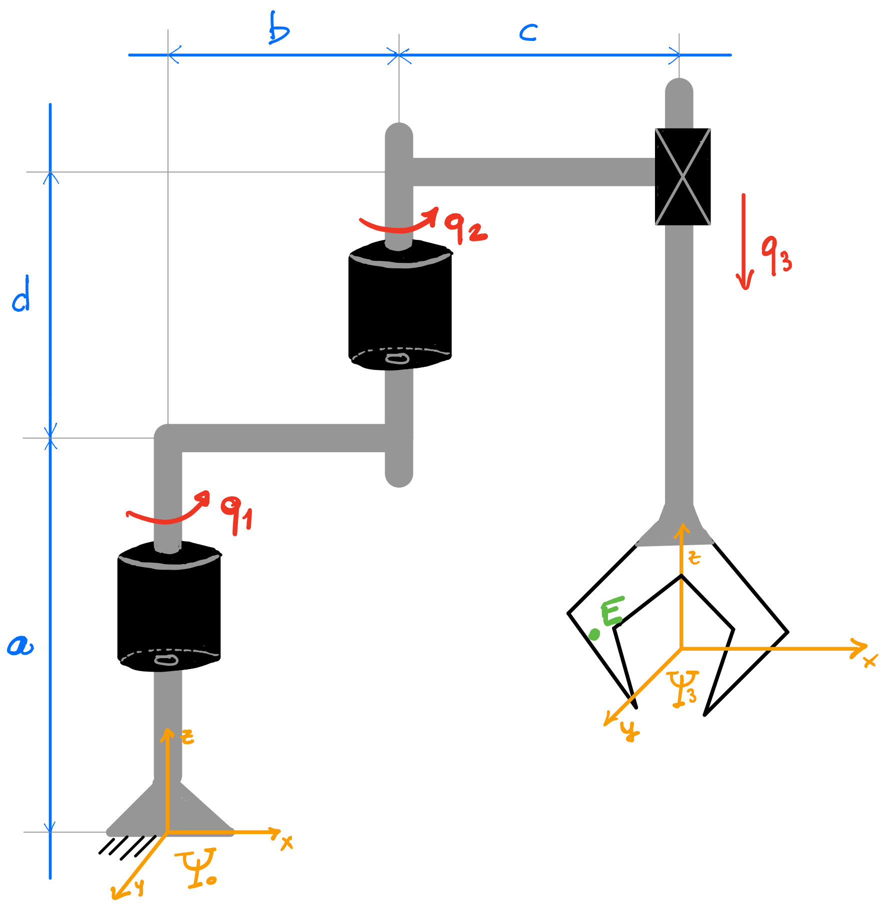
23 June 2020

Consider the 3-DoF robot, the frames and the distances represented in the figure. The robot consists of:

- A rotational joint with axis aligned in direction z of the base frame Ψ_0;
- A second rotational joint aligned on a parallel direction to the first;
- A prismatic joint directed along z of the end-effector frame Ψ_3;

The point E is part of the end effector and has constant coordinates $(-1, 0, 0)^T$ in Ψ_3, i.e. has homogeneous coordinates $E^3 = (-1, 0, 0, 1)^T$ in Ψ_3. Consider as reference configuration for the rotational joints ($q_1 = q_2 = 0$) the one represented in the figure, and make a choice for the reference configuration of the prismatic joint (i.e. the configuration corresponding to $q_3 = 0$).

1. Give Brockett's equation for direct kinematics and all unit twists in reference configurations that you need to calculate $H_3^0(q)$. *Note: There is no need to write out the matrix exponentials but you must calculate the vector products that appear inside your expressions.* (1 Point)
2. Compute the geometric Jacobian $J(q)$ such that $T_3^{0,0} = J(q)\dot{q}$. *Note: You must calculate the vector products that appear inside your expressions.* (2 Points)
3. Compute the velocity of the point E in the inertial frame Ψ_0 as function of q and $\dot{q}$. (0.5 Points)
4. Suppose an external wrench is applied on the end effector with coordinates in Ψ_3 given by W^3=(0 0 0 1 0 0), i.e. the wrench is constant in the end-effector frame. This could be the case if e.g. a propeller mounted of the end-effector would produce a constant thrust. Compute the generalised forces, as function of q and $\dot{q}$, which are experienced by the joints as effect caused by this wrench. (0.5 Points)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