1a Since $\mathfrak{se}(3)$ is a Lie algebra, it possesses an antisymmetric operator called the commutator given by $[\tilde{T}_1, \tilde{T}_2] = \tilde{T}_1\tilde{T}_2 - \tilde{T}_2\tilde{T}_1$ with $\tilde{T}_i \in \mathfrak{se}(3)$.
What is true about the commutator? (Multiple answers may be correct.)

- [x] $[\tilde{T}_1, \tilde{T}_2] \in \mathfrak{se}(3)$
- [ ] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies $\omega_1 = \omega_2 = 0$
- [ ] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies that the matrices $\tilde{T}_1, \tilde{T}_2$ commute
- [ ] $[\tilde{T}_1, \tilde{T}_2]$ is a wrench
- [ ] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies $\omega_1 \wedge \omega_2 = 0$
- [ ] $[\tilde{T}_1, \tilde{T}_2] = 2[\tilde{T}_2, \tilde{T}_1]$

[x] $[\tilde{T}_1, \tilde{T}_2] \in \mathfrak{se}(3)$	0.66
[ ] $[\tilde{T}_1, \tilde{T}_2]$ is a wrench	-0.66
[ ] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies $\omega_1 = \omega_2 = 0$	-0.66
[ ] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies $\omega_1 \wedge \omega_2 = 0$	0.66
[x] $[\tilde{T}_1, \tilde{T}_2] = 0$ implies that the matrices $\tilde{T}_1, \tilde{T}_2$ commute	0.67
[ ] $[\tilde{T}_1, \tilde{T}_2] = 2[\tilde{T}_2, \tilde{T}_1]$	-0.67

Messages

1p **1b** Consider a rotating rigid body that is pinned from its center. Attached to the rotating body is the body-fixed frame Ψ_B attached to the center. Now consider another fixed-intertial frame Ψ_I with the same origin as Ψ_B. The configuration of the body is described by R_B^I.
Let p be a point attached to the rotating body (i.e. $\dot{p}^B = 0$).
What is the velocity of the point p expressed in Ψ_I?

- ( ) $\dot{p}^I = \omega_I^{I,B} \wedge p^I$
- ( ) $\dot{p}^I = \omega_I^{B,B} \wedge p^I$
- (x) $\dot{p}^I = \omega_B^{I,I} \wedge p^I$
- ( ) $\dot{p}^I = \omega_B^{B,I} \wedge p^I$

( ) $\dot{p}^I = \omega_I^{I,B} \wedge p^I$	0.0
(x) $\dot{p}^I = \omega_B^{I,I} \wedge p^I$	1.0
( ) $\dot{p}^I = \omega_I^{B,B} \wedge p^I$	0.0
( ) $\dot{p}^I = \omega_B^{B,I} \wedge p^I$	0.0

1c Consider the Lie group SE(3). What is true about its tangent space TSE(3)? (Multiple answers can be correct.)

- [ ] It is the space of generalized velocities, T
- [ ] The time derivative of homogeneous matrices live in this tangent space.
- [ ] In the identity of SE(3), it is the Lie algebra $\mathfrak{se}(3)$
- [ ] By multiplying the elements of $T_H\mathrm{SE}(3)$ by H^{-1}, one obtains T.

[ ] It is the space of generalized velocities, T	-1.0
[ ] In the identity of SE(3), it is the Lie algebra $\mathfrak{se}(3)$	1.0
[ ] The time derivative of homogeneous matrices live in this tangent space.	1.0
[ ] By multiplying the elements of $T_H\mathrm{SE}(3)$ by H^{-1}, one obtains T.	-1.0

2p **2a** What is a proper implementation for the function "f"?
("tilde" is a function that gives the tilde form of a vector twist; "Adjoint" is a function returning the Adjoint of an H-matrix.)

- ○ expm(Ti*qi)
- ○ expm(tilde(Ti)*qi)
- ● expm(Adjoint(Hi0)*Ti*qi)
- ○ exp(Ti*qi)
- ○ exp(tilde(Ti)*qi)
- ○ exp(Adjoint(Hi0)*tilde(Ti)*qi)

Question 2a was incorrect, right answer is
expm(tilde(Adjoint(Hio)*Ti)*qi)

2b The matrix exponential is a map from $se(3)$ to $SE(3)$. If we want to obtain $H_A^B(t) = e^{Xt} H_A^B(0)$, what should be in the place of X?

- ○ $X = \omega_A^{B,B}$
- ○ $X = \tilde{T}_A^{B,B}$
- ○ $X = T_A^{B,A}$
- ○ $X = \tilde{\omega}_B^{A,A}$
- ○ $X = \tilde{T}_B^{A,A}$
- ● $X = T_A^{B,B}$

○ $X = \omega_A^{B,B}$	0.0
○ $X = \tilde{\omega}_B^{A,A}$	0.0
○ $X = \tilde{T}_A^{B,B}$	1.0
○ $X = \tilde{T}_B^{A,A}$	0.0
○ $X = T_A^{B,A}$	0.0
◉ $X = T_A^{B,B}$	0.0

1p **2c** The dynamics equation for a single rigid body is given by:
$(\dot{\mathcal{P}}^k)^\top = \text{ad}_{T_k^{k,0}}(\mathcal{P}^k)^\top + (W^k)^\top$,
where $T_k^{k,0}$ is the twist of the body.
What is $\mathcal{P}^k$?

- ○ The momentum co-vector expressed in an inertial reference frame.
- ○ The momentum co-vector expressed in a body-fixed reference frame.
- ○ The momentum vector expressed in an inertial reference frame.
- ○ The momentum vector expressed in a body-fixed reference frame.

○ The momentum co-vector expressed in an inertial reference frame.	0.0
○ The momentum co-vector expressed in a body-fixed reference frame.	1.0
○ The momentum vector expressed in an inertial reference frame.	0.0
○ The momentum vector expressed in a body-fixed reference frame.	0.0

2p **3a** What is required for practical implementation of computed torque/feedback linearisation control?

☐	The dynamic matrices M, C, G (of eq. (10) in the formula sheet) must be precisely known.	☐	The dynamic matrices M, C and G must all be invertible.
☒	An accurate measurement of all joint positions must be available.	☐	An accurate measurement of all joint velocities must be available.
☐	An accurate measurement of all joint accelerations must be available.	☐	All joints must be torque-controlled.
☐	All joints must have well-tuned local PID controllers.		

☐	The dynamic matrices M, C, G (of eq. (10) in the formula sheet) must be precisely known.	0.5
☐	The dynamic matrices M, C and G must all be invertible.	-1.0
☑	An accurate measurement of all joint positions must be available.	0.5
☐	An accurate measurement of all joint velocities must be available.	0.5
☐	An accurate measurement of all joint accelerations must be available.	-1.0
☐	All joints must be torque-controlled.	0.5
☑	All joints must have well-tuned local PID controllers.	-1.0

Messages

1p **3b** If you want to do control by interconnection as discussed in the last lecture, which matrices (of the robotics equation) do you need to know?

○	All of M, C and V (or G)	○	Only M and C
○	Only V (or G)	○	None

○	All of M, C and V (or G)	0.0
○	Only M and C	0.0
○	Only V (or G)	1.0
○	None	0.0

Messages

4p **4** A rigid body motion with respect to an inertial frame Ψ_i can be described by a time dependent function $H_j^i(t) \in \mathrm{SE}(3)$, being Ψ_j a body-fixed frame. Its time derivative $\dot{H}(t) \in T_H\mathrm{SE}(3)$ can be associated to a twist by means of a left $(H^{-1}\dot{H})$ or right $(\dot{H}H^{-1})$ transport.
Describe the structure of a twist and assign proper indices i, j in the two cases, motivating the choice.
Write explicitly the time evolution of the coordinates of a point fixed in Ψj with respect to Ψi in the two cases. In the final expressions the elements ω and v in the twist have to appear.
Hint: Start from the time derivative of the coordinate of a body-fixed point, expressed in the inertial frame.

Write twist with correct indices in 2 cases (L&R), highlighting they belong to se(3).	1.0
Motivating through computation that right is th twist in spatial and left in body frame.	1.0
Split the twist in w and v in the two cases.	1.0
Explain the role of w and v in the two cases.	1.0
None of the above	0.0
Specific adjustments	1.0

4p **5** Computed torque control, or feedback linearisation, is a control method that "cancels" all the dynamics of a robotic manipulator and makes it behave according to desired "error dynamics", to follow a desired path.
Given the dynamic equation of a manipulator not touching the environment, without friction:
$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau^\top$
and the desired error dynamics
$\ddot{e} + K_v\dot{e} + K_p e = 0$ (with $e := q - q_d$)
with K_v and K_p still-to-be-chosen controller parameters, and reference signal $q_d(t)$ known a priori.
Question: derive the model-based control law of the form $\tau^\top = f(q,\dot{q})$ that achieves the desired dynamics.
Note: $\tau^\top$ is a function of the measured q and $\dot{q}$, but may also depend on system parameters M, C, G, the known signal q_d and the controller parameters K_v, K_p.

Feedback linearisation: putting C and G in f	1.0
Feedback linearisation: adding $M(q) \cdot \ddot{q}_d$ to f	1.0
Pole placement: rewriting error dynamics to obtain expression for $\ddot{q} = \ddot{q}_d - K_v\dot{e} - K_p e$.	1.0
Finalising: rewriting $\ddot{q}$ to u and adding pole placement, or different substitution to same effect.	1.0
Dividing by $M(q)$ without stating this is possible because M is positive definite as mass matrix.	-1.0
None of the above	0.0
Specific adjustments	0.0

6p **6a** Give Brockett's equation for direct kinematics and all unit twists/reference configurations that you need to calculate $H_3^0(q)$.
You must give twists in vector form; there is no need to write out matrix exponentials. You must complete vector products, if any, that appear in unit twists. *In other words: for the twists we want to see just 6 numbers each.*

T_1: right $\omega = (0 \quad 0 \quad 1)^\top$ and $r = (0 \quad 1 \quad 0)^\top$.	1.0
T_1: right $\lambda = \frac{0.1\,\mathrm{m}}{2\pi} \approx 0.016\ \mathrm{m}$	1.0
T_1: right calculation of $v = r \wedge \omega + \lambda v = (1 \quad 0 \quad 0.016)^\top$.	1.0
$T_2 = (0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0)^\top$	1.0
$T_3 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1)$	1.0
$R_3^0(0) = I_{3\times3}; \quad p_3^0 = (0 \quad 1 \quad 0)$.	1.0
None of the above	0.0
Specific adjustments	0.0

6p **6b** Give an expression for the Geometric Jacobian $J(q)$.
You must write out any vector products that may appear in your expressions; again we want to see 6 numbers for each unit twist that appears in J.
Do not forget to verify your unit twists in some way, showing why your expression is correct or, if you're unlucky, incorrect.

T_1: right $\omega = (0 \quad 0 \quad 1)^\top$ and $r = (0 \quad 1 \quad 0)^\top$.	1.0
T_1: right $\lambda = \frac{0.1\,\mathrm{m}}{2\pi} \approx 0.016\ \mathrm{m}$	1.0
T_1: right calculation of $v = r \wedge \omega + \lambda v = (1 \quad 0 \quad 0.016)^\top$.	1.0
$T_2 = (0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0)^\top$	1.0
$T_3 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1)$	1.0
$R_3^0(0) = I_{3\times3}; \quad p_3^0 = (0 \quad 1 \quad 0)$.	1.0
None of the above	0.0
Specific adjustments	0.0