Written (re^{2}sit) Exam

Modern Robotics

23 September 2015, 9:00 – 12:00

Read the following VERY carefully.

- The exam is composed of 2 Parts: theory and exercises. The first part is *without* any books or notes and it will last for 2 hours max. At the end of the 2 hours, your paper will be collected and the second part of 60 minutes will start. You are allowed to use any notes, printed sheets, etc. for the second part. In summary:
  - Part I: 2 hours max, no books, hand in when you're done.
  - Part II: 60 minutes, with books.

  Please make sure to start Part II on a new page, so you can hand in Part I separately!
- If you finish and hand in the first part sooner than after 2 hours, you may of course start working on the second part, *without your notes.* Only when *everyone* has handed in the first part, books and notes are allowed. (The supervisor will indicate this.)
- If the answers are not easily readable, the corresponding answer will be given 0 points. Therefore, *write clearly.* If necessary, use A and B instead of i and j.
- Read every question well before answering.
- Write ALL your reasoning steps on paper.
- Write your name and student number clearly legible on EACH piece of paper.
- Good luck!

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Part I: Theory (Closed Book)

In this part of the exam, you will try to derive the expression for the Geometric Jacobian $\mathbf{J}$, which maps a serial chain's joint velocities $\dot{\boldsymbol{q}}$ to the end-effector twist $T_n^{0,0}$ (1).

$$\mathbf{J} : \mathcal{TQ} \rightarrow \mathfrak{se}(3) \; ; \quad \dot{\boldsymbol{q}} \mapsto \mathbf{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{1}$$

Question 1 (weight: 1)

Without loss of generality, assume that our serial-chain robot arm has 3 joints. Starting from the definition of $\tilde{T}_3^{0,0}$, show that the end-effector twist $T_3^{0,0}$ is the sum of all individual joint twists.

Important note: we will be very strict when it comes to notation here and after, so take care of the indices and the tildes.

Question 2 (weight: 2)

There are three types of joints: a rotational, a translational and a screw joint. Each of these joints or kinematic pairs can be described with a constant unit twist $\hat{T}$:

$$T_{i+1}^{i,i} = \alpha \hat{T}_{\mathrm{i+1}}^{\mathrm{i,i}}. \tag{2}$$

For each of the three joint types, give the form of the unit twist $\hat{T}_{\mathrm{i+1}}^{\mathrm{i,i}}$ and *briefly* explain what the symbols mean.

Example: Suppose there is a fourth type of kinematic pair, the "magic joint." Your answer could be:

"magic joint: $\hat{T}_{i+1}^{i,i} = \begin{pmatrix} r \\ \hat{q} \end{pmatrix}$, where r is the vector pointing from frame the joint rotation axis to i and $\hat{q}$ is the unit-length vector of the magic direction."

Question 3 (weight: 3)

We will need to express each joint's twist $T_{\mathrm{i+1}}^{\mathrm{i,i}}$ as a function of its joint velocity $\dot{q}_{i+1}$. It turns out that this is a simple, constant map $\hat{T}_{i+1}^{i,i}$, as in (3):

$$\hat{\boldsymbol{T}}_{i+1}^{i,i} : \mathcal{TQ} \rightarrow \mathfrak{se}(3) \; ; \quad \dot{q}_{i+1} \mapsto \hat{\boldsymbol{T}}_{i+1}^{i,i} \; \dot{q}_{i+1} \tag{3}$$

Using the exponential map of $\hat{T}_{\mathrm{i+1}}^{\mathrm{i,i}}$ (the constant unit twist), show that (3) is valid, i.e., show that $T_{\mathrm{i+1}}^{\mathrm{i,i}} = \hat{T}_{i+1}^{i,i} \cdot \dot{q}_{i+1}$.

Some hints:

1. If you don't remember what the exponential map of a twist is, start from the definition of $\tilde{T}_{\mathrm{i+1}}^{\mathrm{i,i}}$, rewrite it to a differential equation of the form $\dot{\boldsymbol{x}} = \boldsymbol{x} \cdot \mathbf{A}$ and find its solution $\boldsymbol{x} = e^{\mathbf{A}t} \cdot \boldsymbol{x}(0)$; then replace t by q_{i+1}.
2. As always, the definition of $\tilde{T}_{\mathrm{i+1}}^{\mathrm{i,i}}$ is a good place to start.
3. You may use 2 and 1 for $i+1$ and i, respectively, to make your derivation more concise.

Part II: Exercises (Open Book)

Question 4 (weight: 4)

For this extra resit, we have an extra intelligent crane: it has only 3 joints, but can still reach everywhere in the workspace. Please study Figure 1 carefully and read the description below.

- The first joint is a screw-type joint with a pitch of 10 cm/rev; that is: for each full rotation of the boom head, it travels 0.1 m upwards (in $+\hat{z}$). The joint coordinate is q_1 [rad], positive counter-clockwise (see the top view drawing).
- The second joint is a translation-type joint, moving a pulley along the boom. The joint coordinate is q_2 [m].
- The third joint is a translation-type joint modelling the cable, so downwards from the pulley. The joint coordinate is q_3 [m].
- The reference frame Ψ_0 is attached to the base with $\hat{z}$ pointing up. Ψ_1 is attached to the boom head with $\hat{x}$ pointing along the boom; Ψ_2 to the pulley with $\hat{x}$ pointing along the boom and Ψ_3 to the hook with $\hat{x}$ pointing along the boom. All $\hat{z}$ point upwards.
- When all joint positions are 0, the boom head is fully lowered ($z^0_{\text{head}} = 0\,\text{m}$) and the boom points along $+\hat{x}$ of Ψ_0; the pulley is at the boom head ($x^1_{\text{pulley}} = 0\,\text{m}$) and the hook is fully retracted ($z^2_{\text{hook}} = 0\,\text{m}$).

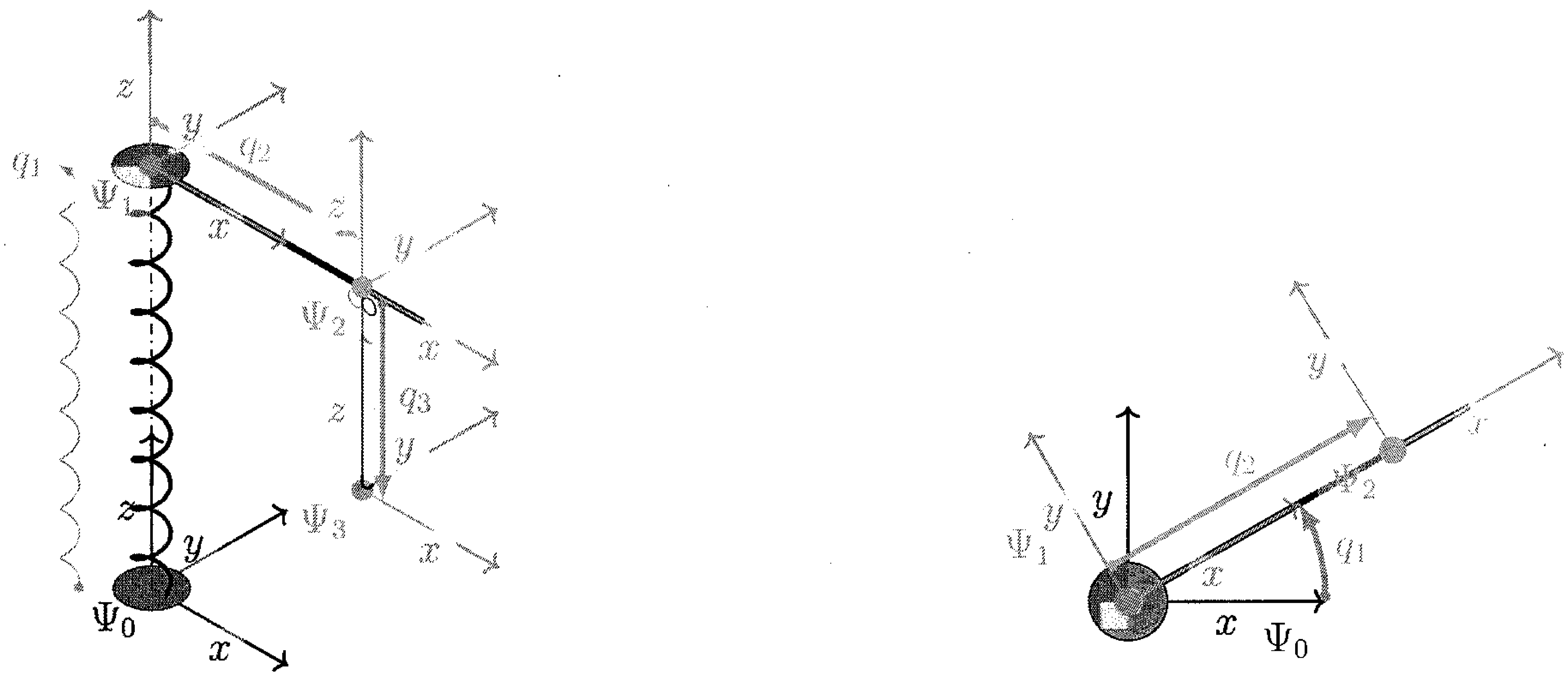

Figure 1: The serial-chain crane under scrutiny in question 4: 3-D view (left) and top view (right).

Question 4A) Give an expression for $H^0_3(\boldsymbol{q})$ using Brockett's direct kinematics formula. There is no need to give the tilde forms of unit twists, nor to write out matrix exponentials.

Question 4B) Give an expression for the Geometric Jacobian $\mathbf{J}(\boldsymbol{q})$, such that

$$T_4^{0,0} = \mathbf{J}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}}.$$