Written Exam
Modern Robotics

On Friday, 1st of July from 9:00 AM to 12:00 AM

Read the following VERY carefully.

- The exam is composed of 2 Parts: theory and exercises. The first part is WITHOUT any books or notes and it will last 2 hours. At the end of the 2 hours, the exam will be collected and after 15 minutes the second part of 45 minutes will start. You are allowed to use anything you want for the second part:
  - Part I, 2 hours, no books, hand in at the end.
  - Break, 15 minutes, collect books.
  - Part II, 45 minutes, with books.
- If you are ready with the first part, you can start working on the second part, BUT you obviously need to hand in the first part before doing so, due to the fact that you can use the notes for the second part.
- NEVER talk with your neighbor.
- If the answers are not easily readable, the corresponding answer will be given 0 points. Therefore WRITE CLEARLY.
- Read every question well before answering.
- Write ALL your reasoning steps on paper.
- Write your name and student number clearly readable on EACH piece of paper.
- Good luck !

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Part I: theory

(Closed Book)

Question 1

(weight:1)

Given: We have seen that for the coordinate change of twists the following holds:

$$T_i^{k,j} = Ad_{H_l^k} T_i^{l,j}$$

Asked: Give an expression for the coordinate change of wrenches between Ψ_l and Ψ_k and prove it. Make explicit all assumptions!

Question 2

(weight:3)

Given: Given a manipulator with n degrees of freedom (d.o.f.), choose a frame i rigidly attached to link i for $i = 1, \ldots, n$. Frame Ψ_0 is assumed to be fixed with the base of the robot. We have seen that the relation between the joints speed and the end effector twist is given by:

$$T_n^{0,0} = J(q)\dot{q}$$

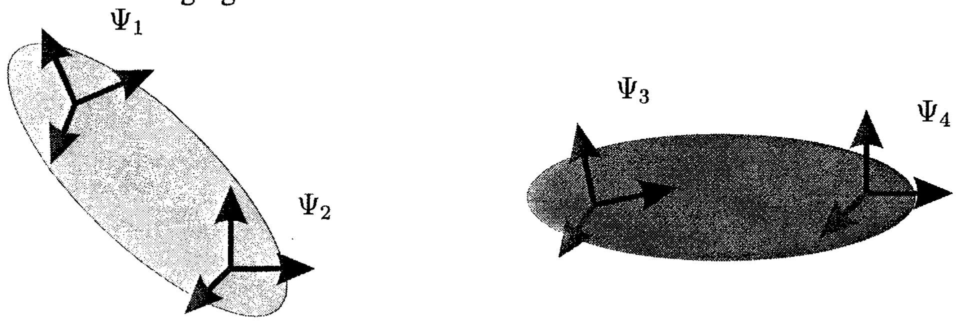
where $J(q)$ is called the geometric Jacobian.

Asked: Show ALL the steps to get to an expression of the Jacobian. As a tip, start considering the chain rule $H_n^0 = H_1^0 H_2^1 \ldots H_n^{n-1}$.

Question 3

(weight:4)

Given: Consider two bodies with two frames rigidly connected on each of the bodies as reported in the following figure:



Asked: Prove the following identities:

- $T_4^{1,1} = T_3^{1,1}$
- $T_3^{2,1} = T_3^{2,2}$

Part II: exercises

(Open Book)

Question 4

(weight:4)

Given: The following robot is a two degrees of freedom robot. It is composed of 2 links and 2

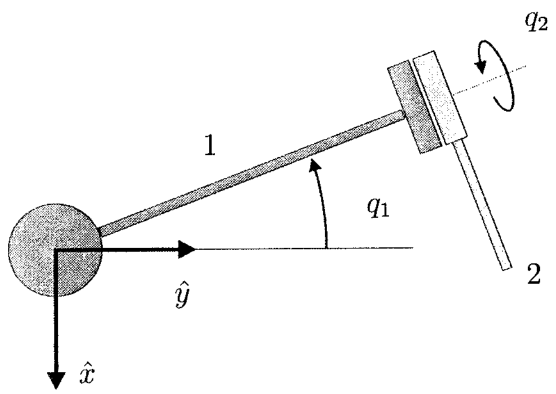


Figure 1: Top View

rotational joints. The first joint constrains the motion between the ground and the first link (dark): the link can rotate around an axis coming out of the sheet of paper and corresponding to the $\hat{z}$ of the drawn coordinate system which is fixed with the ground. The second joint constrains the motion between the first and the second link (light). The axis of rotation is shown in the figure. The scalars q_1 and q_2 give the respective relative angles as shown in the figure.

Asked: Give an expression of the direct kinematics $H_2^0(q_1, q_2)$ considering that:

- Ψ^0 is the frame represented in the figure which is fixed with the world
- Ψ^2 is a frame fixed with body 2 and for which 1.the rotation axis of q_2 corresponds to the z of the frame (pointing outward) 2. the link stick 2 corresponds with the y axis. The distance between the origin of Ψ^0 and Ψ^2 is l.
- $q_1 = q_2 = 0$ corresponds to the first link horizontal and aligned with the $\hat{y}$ axis of Ψ_0 and link 2 is pointing in the direction of $\hat{x}$.